Exercise: Draw HR Diagram of Star Cluster

Guide for Teachers

INDEX   1. Purpose
               2. Contents
               3. Outline
               4. Background
               5. Preparation and Class Plan
               6. Comments
               7. Example for Answers to the Worksheets
               Appendix

1. Purpose

Draw HR diagrams of stars cluster using PC, from FITS-formatted raw frames taken by Hubble Space Telescope, Subaru Telescope (in Hawaii), and Kiso Schmidt Telescope (in Japan). Then do following activities:

- Estimate an age based on HR diagram of globular star cluster.
- Find out the main sequence in HR diagram of open star cluster.
- Discuss about stellar evolution, by drawing HR diagrams of globular and open star clusters.

2. Contents

- Draw HR diagram (in fact, it is color-magnitude diagram; CM diagram) of star cluster from archival data of Hubble Space Telescope (HST) and Kiso Observatory.
- Discuss about stellar evolution on the CM diagram.
- Compare the drawn CM diagram with theoretical curves, and consider age of the star cluster.
- Draw CM diagrams of several star clusters, and compare the ages. [optional]

Things to be prepared:
- Image processing software 'Makali`i'
- B and V band FITS images of globular and open clusters
- Whole-area images of the globular and open clusters
- MS Excel(98,2000,2002)
- Excel macro for drawing the CM diagram

3. Outline

1. Flow of work, explanation of data, and explanation of software
2. Photometry
3. Arrangement of the data, drawing HR (CM) diagram
4. Check and discussion

4. Background

4.1. HR Diagram

HR diagram, Heltzsprung -Russell diagram, designed by E. Heltzsprung and H. N. Russell, is a diagram in which the star data are plotted, with its horizontal axis meaning spectral type, and its vertical axis meaning absolute magnitude.

4.2. Color-Magnitude Diagram of Star Cluster

You can think that stars in the cluster are at nearly the same distance from the Sun. Therefore, you can draw a plot that has the same meaning as the HR diagram, using apparent magnitude instead of absolute magnitude for the vertical axis.

It is not easy that you draw an HR diagram after getting spectrum of each star in the cluster. Thus, astronomers usually draw a color-magnitude diagram (CM diagram) using color index (for example, B-V index) instead of spectral type for the horizontal axis.

4.3. Color Index and Spectral Type

4.3.1. Color index

A ratio of brightness in two different wavelengths for a star depends on its surface temperature (that is, spectral type).

Fig.1. shows the difference of temperature of black-body spectrum and difference of 'color'. Vertical axis is shown in log scale. Blue, yellow, and red curves indicate black-body spectra with temperatures of 12000, 6000, and 3000 K.

Shown in Fig. 1, consider a ratio of brightness that is taken through two filters that transmit

different wavelengths. The black-body spectrum has shorter peak-wavelength of emission for higher temperature. Therefore, for the black-body spectra with different temperatures, when you measure the ratio of brightness in two different wavelengths, it is brighter at shorter wavelength than at longer wavelength for the higher temperature.
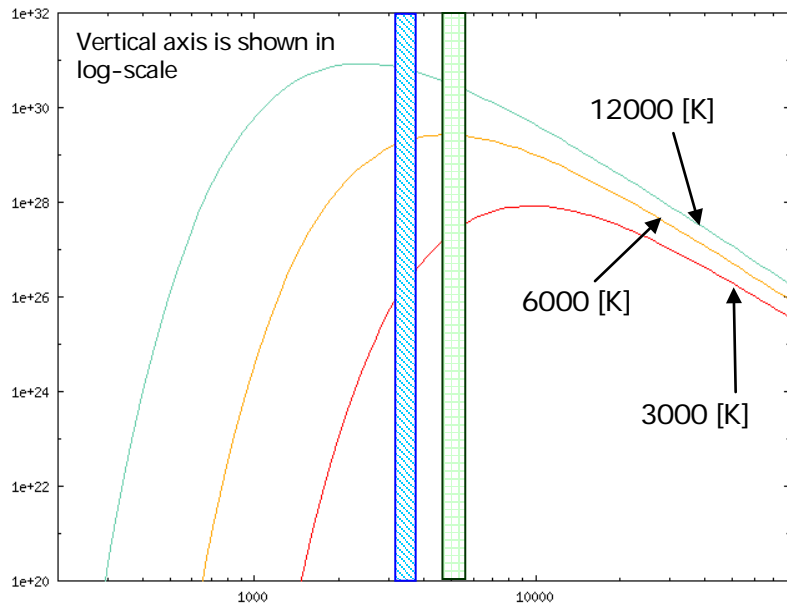
Fig1. Difference of temperature of black-body spectrum and difference of 'color'. Vertical axis is shown in log scale. Blue, yellow, and red curves indicate black-body spectra with temperatures of 12000, 6000, and 3000 K.

The case for the stars is the same. Consider that you measure brightness of star through filters that transmit different wavelengths. Surface temperature of star ranges from around 3000 to several tens of thousands degrees in Kelvin. For example, in the optical wavelength range (roughly from 400 to 800 nm), a star with higher temperature is brighter in shorter wavelengths than in longer wavelengths. When brightness is expressed in magnitude scale, the ratio of brightness corresponds to difference in magnitude. The difference in magnitude is called 'color index'.

In this workbook, we use B-band filter that transmits blue lights for the shorter-wavelength filter, and V-band filter that transmits green lights for the longer-wavelength filter. Brightness in B-band image compared to that in V-band one is larger for a higher-temperature star. The ratio of B- and V-bands images is derived by the difference in magnitude, B-V. In this workbook, we use B-V for the color index (note: B is for blue, V is for visual).

4.3.2. Relation between Color Index and Spectral Type

Spectral type of a star is determined by appearance of absorption lines in spectrum as described in table 1. These features are determined by stellar surface temperature. Table 2 shows the relation between the spectral type and the color index (B-V).

Table 1. Characteristics of Spectral Types.

Sp. type	Eff. temp. [K]	Characteristics
O	45000- 32000	Ionized helium; highly-ionized oxygen, nitrogen, and carbon are seen.
B	30000-10000	Hydrogen absorption lines are stronger. Neutral helium lines are strongest.
A	9700-7500	Hydrogen absorption lines are strongest. Ionized metallic lines are getting stronger.
F	7300-6000	Hydrogen absorption lines are getting weaker. Ionized calcium absorption lines (H and K lines), and metallic absorption are getting stronger.
G	6000-4700	Strong absorption lines of ionized calcium (H and K lines) and metallic absorption lines are seen. Hydrogen absorption lines do not stand out.
K	4700-4000	Strong and broad absorption lines of ionized calcium lines (H and K lines) are seen. Metallic absorption lines are blended each other.
M	4000-3500	Titanium oxide absorption band stands out.
R, N	3000	Carbon and cyanogen absorption band stands out.
S	3000	Zirconium oxide absorption band stands out.

Table 2. Relation between spectral type and (B-V) color index.
(for main-sequence stars)

Spectral type	Color index (B-V)
O5	-0.3
B0	-0.3
B5	-0.16
A0	0.00
A5	+0.15
F0	+0.33
F5	+0.45
G0	+0.60
G5	+0.68
K0	+0.81
K5	+1.15
M0	+1.4
M5	+1.6

4.4. CM Diagram and Stellar Evolution, Star Cluster's Evolution

All stars in a cluster are considered to have approximately the same age. Based on this fact, we can read the age of the cluster from the HR or CM diagram of the cluster.
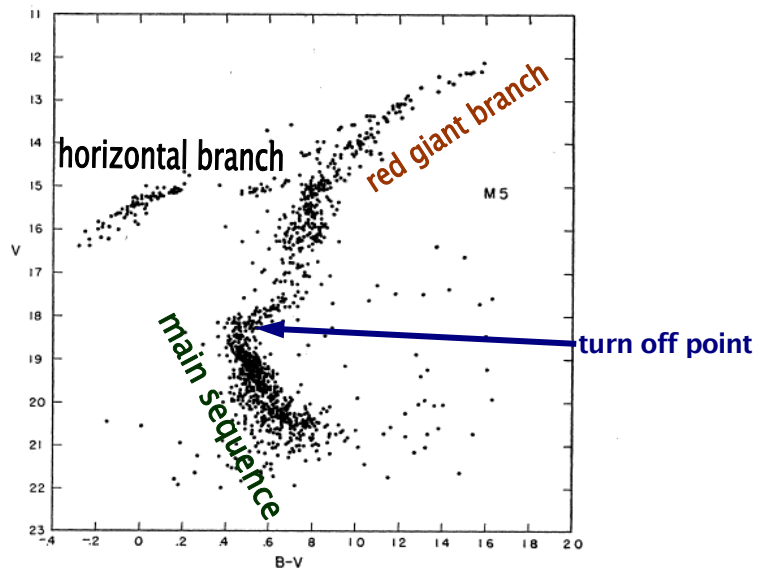

FIG. 3.—The color-magnitude diagram for M5. Stars brighter than $V = 16$ (Table 3a) are predominantly transformed from Arp (1955b). Fainter stars (Tables 3b, 3c, 3d) are from present photographic measures.

Fig.2. An example of the CM diagram (Arp et al. 1962, ApJ, Vol.135, p.311).

Main Sequence

A band from the upper left to the lower right in the CM diagram is called main sequence, and stars in the band are called main-sequence stars.

In the core of a main-sequence star, hydrogen atoms are turning into helium atoms through nuclear fusion. The main-sequence stars are stable stars that shine through this nuclear fusion energy.

The more massive a star is, the higher its temperature is, and the brighter the star is. The more massive star evolves faster, so its life time is shorter. Then, stars at upper left of the main sequence are more massive and live shorter lives.

Turn-Off Point

Just after the birth of a cluster, stars with various masses shine as main-sequence stars. However, as time goes by, massive stars at upper left of the main sequence near the ends of their lives, and they turn into red giant stars. In the CM diagram, they move toward right.

The CM diagram of a star cluster with some age has a 'turn' in the main sequence. At that point, the star is just about to leave the main-sequence stage for the red-giant stage. The point is called turn-off point.

Red-Giant Branch

A star shines almost stably with nuclear energy, burning hydrogen into helium. However, when about 10 % of hydrogen in the core has become helium, the core shrinks, and outer layers expand. That makes the star a red giant. In the CM diagram, evolution from main sequence to red giant appears a nearly horizontal shift (after entering the red-giant stage, evolutionary path

goes upward). As the cluster ages, more massive, brighter, and short-lived stars (stars that once plotted at upper left part of the main sequence in the CM diagram) move to the right, the red giant branch develops in the CM diagram which stretches to the upper right.

Horizontal Branch

As hydrogen in the core runs out, the core shrinks, and the star becomes red giant. When the core shrinks even more, temperature goes up farther and helium begins to burn (nuclear fusion of helium). Massive stars start the process of burning helium on the way of the evolution to the red giant. On the other hand, less massive stars like the Sun (less than several times the mass of the Sun), the helium burning starts all at once, then the burning continues steadily. At earlier of this evolutionary stage, the star does not change its brightness much, and moves leftward in the CM diagram, that is, the temperature is getting higher. Then again, it moves to the upper right. We see horizontal branch stars, which stays leftward temporarily when they burn their helium.

Isochrone

In the CM diagram of a cluster, you see a curve that depends on the age, shaped by stars having their own lives depending on the mass. Evolutionary path of a star in the CM diagram can be derived theoretically with a parameter of the stellar mass (in fact, abundance of elements heavier than hydrogen in the outer layers of a star changes the path). For stars of various masses with the same age, connecting positions on the evolutionary paths makes the theoretical CM diagram of a cluster for that age. This curve is called isochrone.

Estimate of Age of Cluster

As mentioned, as time passes after the birth of a cluster, the main-sequence stars change into red giant in order of stellar mass. In the CM diagram of a cluster with a given age, position for a star that ends its main-sequence life time corresponds to the turn off point. Therefore, if you draw a CM diagram and find out the turn-off point, you can consider main-sequence life time of the star at that point as the age of the cluster.

4.5. Photometric Values and Star's Magnitude

Brightness of stars is often expressed using magnitude system. On the other hand, when you make photometry for a star in a FITS data, you will get total count in the region where star lights spread (see later). Therefore, it is necessary to convert the count value measured in the FITS data to the magnitude scale.

Relation between star's brightness and magnitude

Brightness of an object is expressed in magnitude. There are relations between the brightness and magnitude as:

- brighter object has smaller value of magnitude.
- ratio of brightness is expressed as difference in magnitude.
- a brightness ratio of 100 corresponds to a magnitude difference of 5.

For example, a ratio for brightness B_1 and B_2 is expressed as a difference of magnitudes between m_1 and m_2 as a following equation:

$$m_1 - m_2 = -2.5 \log (B_1/B_2).$$

The count value in the FITS data is proportional to the brightness (though we should take into account many things such as condition of observations, inhomogeneity in sensitivity of camera, here we think we have data with all necessary corrections). Therefore, above equation can be rewritten using the count values (put C_1 and C_2, respectively) as follows:

$$m_1 - m_2 = -2.5 \log (C_1/C_2).$$

In this work, you will derive the color index using counts in B-band and V-band images (put C_B and C_V, respectively). Thus:

$$B\text{-}V = m_B - m_V = -2.5 \log (C_B/C_V).$$

In analyzing the real data, you should also adjust the filter transmission curve to the standard filter system. Usually, astronomers observe the standard stars the magnitudes of which are measured in the standard system, then calibrate the brightness of stars. In this workbook, we prepared images that are converted into the standard system. So, you do not need such calibration.

Magnitude (apparent magnitude) and absolute magnitude

Brightness we see of an object varies depending on distance. The nearer the object is, the brighter we see. Therefore, brightness observed from a site (usually, consider Earth) is called apparent brightness (apparent magnitude). On the other hand, astronomers define absolute magnitude as an index of intrinsic brightness, independent of distance. For stars, it is defined as

an apparent magnitude at a distance of 10 pc.

Put apparent magnitude of a star at a distance of *d* pc from the Sun, *m*, and put absolute magnitude, *M*, and if we can ignore interstellar absorptions, then we have a relation:

$m - M = 5 \log d - 5$.

Magnitude used in HR (CM) diagram of star cluster

Usually, the vertical axis of the HR diagram indicates absolute magnitude. On the other hand, for objects the distance to them can be considered nearly the same, such as stars in a star cluster or a galaxy, we have a diagram with the same meaning using apparent magnitude instead of absolute magnitude for the vertical axis.

4.6. Processes of FITS Data

FITS data's pixels have values of count that correspond to brightness. Brightness of a star is measured by summing up values of count with a star's region.

FITS data analyzing software 'Makali`i' is software that can see count in the pixel at any position (x, y), integrate counts within any region, and operate counts between frames easily.

In the photometry mode of Makali`i, when you click a star you want to photometry with the cursor on its center, center of star's image is calculated, and Makali`i returns total count integrated in a circle area with a given radius. Besides, an average value at an annular region around the area of the star is subtracted as the background level. The radius of the aperture as the star area is given as FWHM * 1.5 + 0.5 pixels for auto mode (FWHM: full width at half maximal brightness). As the background area, inner radius of the annulus is given FWHM * 3 - 0.5 pixels and width of the annulus is 2 pixels automatically (under the auto mode). Image data taken by CCD camera contains noise and inhomogeneity in sensitivity due to characteristics of the camera. This workbook provides data with proper corrections for that.

5. Preparation and Class Plan

Preparation

1.   Download and install Makali`i.

2. Download FITS data.

3. Download whole-area image of the cluster.

4. Teacher should work through once before the class day, from a point of student's view.

5. If students should have time to get acquainted with handling Makali`i, have some time.

Class plan (example)

1st period: Introduction; explain the work, see image data carefully (or start the photometry)

2nd period: Do photometry earnestly, output to CSV file

3rd period: Convert count values measured from data to magnitudes and colors, draw CM diagram (using Excel macro). Discuss based on the CM diagram.

6. Supplement

Corresponding to the National Curriculum Guidelines in Japan (Heisei 15 version)

> Senior High School
>> Earth Science I
>>> (2) Atmosphere, Ocean, and Universe
>> Earth Science II
>>> (4) Extensive Studies

7. Example for Answers to the Worksheets

7.1. Globular Cluster Part

Q.1. Referring the below figure, write down names to parts in your CM diagram.

It is possible that picking out the turn-off point varies. It is the position where the main sequence leaves. Note that it is not the position where the red-giant branch begins. There may not be other problems.

Q.2. Why do you think are the main-sequence stars rare on the dotted part above the turn-off point?

Because of change in energy generation mechanism, the main-sequence stars are considered to move toward the red-giant branch. We see this because we limit the data only for a cluster.

Q.3. Red-giant branch tells us the evolutionary path of stars in the globular cluster. Write an arrow for the path that you guess. Why can you guess so?

It is possible that many students answer that the main-sequence stars evolve as the red-giant stars along the line where plots are concentrated in the CM diagram. However, it is not correct. The evolutionary path of each star is actually near the theoretical line (see teacher's guide for details).

Q.4. What are horizontal-branch stars?
 Hint: They are relatively stable on their ways through their evolution.

Note that we can not say that they are a group of stars that are changing from main sequence to red giant. They are temporarily shining steadily. We can say that they are at a stage of helium fusion (see teacher's guide for details).

Q.5. How bright is the horizontal branch (in magnitude)? If your classmates investigate other globular cluster, compare your estimation with his or hers. What can you conclude from that?

Let students read absolute magnitude (the vertical-line value) of the horizontal branch in the CM diagram.

Q.6. How bright is the turn-off point from the main sequence to the red giant? Compare results with your classmates. Is there any difference among yours?

Let students read absolute magnitude (the vertical-line value) of the turn-off point in the CM diagram. Depending on their readings, the value of absolute magnitude can vary.

Q.7. The turn-off point from the main sequence to the red giant differs from globular clusters to globular clusters. What makes this difference?

The absolute magnitude (the vertical-line value) for the point differs. In a younger cluster, more stars stays in the main sequence, and the absolute magnitude of the turn-off point is smaller (that is, brighter, and upper in the diagram).

Q.8. Fainter stars below the turn-off point, in the lower part of the diagram, spread like skirt of a mountain. What makes this distribution?

The stars spread into horizontal direction. Fainter stars have poor quality in pictures. Since the color index is calculated from two photometry values, deviation from the true value grows for the color. Besides, since we can not distinguish cluster stars from others, these other stars (we say them background stars) may contaminate the cluster area. The fainter the star is, the bigger the number of stars is, so faint stars spread in the lower area.

Q.9. The position of the turn-off point varies as the globular cluster ages. Consider that (i), (ii), and (iii) in the diagram below are for three clusters. Which one is for the oldest cluster? Why do you think so?

The oldest one is shown by (iii). Because even less-massive stars that are slow in hydrogen fusion and thus slow in leaving the main sequence turned into red giants. Let students understand that duration time of the nuclear fusion (we can say it nearly the life time of a star) is shorter for more massive stars, and that stars plotted in the left of the CM diagram have higher temperatures and larger masses.

Q.10. Estimate the age of the globular cluster by comparing with the theoretical curve of globular cluster's evolution.

An age of about 10 billion years is expected. Recent detailed studies show that the age of this globular cluster is 8.3 billion years old. We do not expect such fine value. Being not 1 billion

years is a good answer.

Q.11. Which is the closest to your estimation for the age? What can you say when you get the information of age of globular cluster?

    (1) Birth of human beings

    (2) Birth of Earth

    (3) Formation of the Galaxy

    (4) Origin of the universe

The answer is (3). (1) is as early as about several million years ago, (2) is about 4.6 billion years ago, and (3) is of order of 10 billion years old because it is considered to have happened 1 billion years after the birth of the universe. (4) is considered to be about 14 billion years old.

7.2. Open Cluster Part

Q.1. Find the main sequence from the printed-out CM diagram. Enclose the main sequence by line with a width as narrow as possible.

Though the width expands in the region fainter than 3 in absolute magnitude, upper part can be encircled by a shape band. At least at the upper part, the main sequence can be enclosed like an example of school textbook's HR diagram. Note that the upper top is just moving to the red-giant branch, so the upper sequence turns to the right a little.

Q.2. What are the characteristics of the main-sequence stars? Explain considering meanings of absolute magnitude in the vertical line and color index in the horizontal line.

Confirm the contents in the school textbook.

Q.3. For the Sun, V=4.85 and B-V=0.65.

    (1) Plot this on the CM diagram.

    (2) What characteristics do stars that reside upper than the Sun's position have? Think it by considering the meanings of brightness and color index.

Let students understand that they are brighter than the Sun, and that they are bluer than the Sun. They are more massive than the Sun as well.

(3) In the diagram, stars that reside lower than the Sun do not concentrate but spread, and the number of stars are not large. What do you think makes this?

This question is the same as Q.8. of globular cluster part.

Q.4. Consider that the shape of the CM diagram is general for all clusters. See the two main sequences shown by a solid and a dotted curves in the diagram below (separated by a gap in the vertical direction). What characteristics of the clusters differ to make this difference?

Apparent brightness is different. The shape of the CM diagram does not change. So, to change the apparent brightness, the distance to the cluster must differ. The B-V indices of cluster stars are ratios of brightness in B and V. So, B-V does not change because B and V magnitudes change similarly as the cluster go farther.

Q.5. Compare the main sequence of the HR (CM) diagram shown in your school textbook with that you found out in Q.1. for NGC 1912.
   (1) What do you notice in the upper part of the main sequence? How is the difference like?

Let students notice that upper top part of the main sequence is turning to the right. See Q.1.

   (2) Why do you think makes this?

See Q.1.

7.3. Open Cluster and Globular Cluster Part

Q.1. Decide which type is your CM diagram, referring diagrams below. Write names to parts in your diagram.

The type B is for a globular cluster, and the type A is for an open cluster.

Q.2. Compare results by classmates who investigated open cluster data or globular cluster data. Exchange comments.
   (1) Mention as many points as possible which are similar to and different from each other.

Overlapping is frequent at lower part, fainter than 4 in absolute magnitude. The difference

between their plots is clear as going upper part.

(2) Compare absolute magnitude of the turn-off point.

It is expected that for an open cluster it is about 0, and for a globular cluster it is fainter than 4.

(3) Consider that the difference in part upper than the turn-off point is the shift in the horizontal direction. What characteristic of stars makes this shift?

The horizontal axis means color index. The right side indicates the redder color.

(4) Since the energy generation system changes as the star ages, a star evolves from a protostar to a main-sequence star, a red-giant star, and a white dwarf. Which do you think is older, the open cluster or the globular cluster? Why do you conclude that?

See answer to Q.9 of globular cluster part.

Q.3. Compare derived CM diagram and theoretical curve. Estimate age of cluster you investigated.
   (1) Estimation of the age.

The estimation is about 10 billion years old for globular cluster. See Q.10 of globular cluster part. It is about 1 billion years old for open cluster. In both cases, if the order is correct, it is a good answer.

(2) Why do you guess so?

It is easy if the diagram happens to be fitted with the theoretical curve. If not so, by interpolating the curves, you can read the value for the age.

Appendix

Flow of a class (plan 1)

[1h] Introduction
  Confirmation about the HR diagram
  Confirmation about the stellar evolution
    (mass and speed of evolution)
      Evolution of star cluster as a whole is shown in the HR diagram.
  About the vertical axis, absolute magnitude, photometry
  About the horizontal axis, color index, photometry of two images

[2h] Photometry using Makali`i
  Explanation about photometry
  Photometry works
  Saving the results, output to CSV file, saving the FITS data

[3h] Drawing Graph
  Showing theoretical curve data
  Instructing about fitting of coordinates of the diagram
  Output to CSV file
    Input to Excel sheet, running the macro, printing out the graph

[4h] Discussion
  Estimation of the age by fitting the theoretical curve
  Presentation by some students
  Discussion about the difference among students

Flow of a class (plan 2)

[Preparation]
Image processing software Makali`i
FITS image of a globular cluster (NGC 362), B#2 & V#2
MS Excel(98,2000,2002)

[Purpose]
 Draw HR diagrams of stars cluster using PC, from excellent FITS-formatted raw frames taken

by Hubble Space Telescope, Subaru Telescope and others. Based on the diagram, derive age of globular cluster and discuss about evolution of the Galaxy.

[Procedure]

    <Photometry>

1. Start the application software Makali`i.
2. Open B and V FITS images.
3. Press Photometry button for the B image. Photometry window pops up. The mode should be changed from semi-auto to auto. Drag the photometry window below to hide. Similarly, do for the V image.
4. Then, pick out the same star in both B and V images and click by turns. Click as many as 50 to a few hundred stars.

   [note for picking out stars]

   (1) Avoid the case that two stars lie in one-click circle. If so, you will get the mixed value.
   (2) Avoid stars that lie near the edge of frame.
   (3) Click by turns the same star in B and V frames.
5. When you want to interrupt the work, be sure that you save in files with new names for B and V images. By saving so, you can resume your work from the previous point.
6. After finishing photometry for all stars, save the photometry data sheet to CSV-formatted files for B and V images, respectively.
7. Quit Makali`i. Makali`i asks the saving name: select the overwrite saving mode.

    <Numerical processes, drawing graphs>